

Cambridge IGCSE™

ADDITIONAL MATHEMATICS Paper 1 February/March 2022 MARK SCHEME Maximum Mark: 80 Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of 9 printed pages.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always whole marks (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Ma | Maths-Specific Marking Principles | | | |
|----|---|--|--|--|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. | | | |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. | | | |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. | | | |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). | | | |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. | | | |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. | | | |

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

answers which round to awrt cao correct answer only dependent dep FT follow through after error ignore subsequent working iswnfww not from wrong working or equivalent oe rounded or truncated rot

SC Special Case soi seen or implied

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| Question | Answer | Marks | Guidance |
|----------|--|-------|---|
| 1 | $9kx + 1 = kx^2 + 3(2k + 1)x + 4$, leading to $kx^2 + x(3-3k) + 3$ [= 0] | M1 | For equating the two equations and attempt to obtain a 3 term quadratic equation equated to zero. |
| | $(3-3k)^2 - (4\times 3k)$ oe | M1 | Dep on previous M mark for attempt to use the discriminant in any form |
| | $3k^2 - 10k + 3$ oe | M1 | Dep on previous M mark for simplification to a 3 term quadratic expression in terms of <i>k</i> |
| | Critical values 3 and $\frac{1}{3}$ | A1 | For both |
| | $\frac{1}{3} < k < 3$ | A1 | Mark the final answer |
| 2 | $x = \frac{-(2\sqrt{3}+5) \pm \sqrt{(2\sqrt{3}+5)^2 - 4(3-5\sqrt{3})(-1)}}{2(3-5\sqrt{3})}$ | M1 | For the use of the quadratic formula |
| | $x = \frac{-(2\sqrt{3}+5) \pm \sqrt{12+20\sqrt{3}+25+12-20\sqrt{3}}}{2(3-5\sqrt{3})}$ | M1 | For expansion of the square root, must see at least 4 terms |
| | $x = \frac{-12 - 2\sqrt{3}}{2(3 - 5\sqrt{3})}$ oe, $x = \frac{2 - 2\sqrt{3}}{2(3 - 5\sqrt{3})}$ oe | A1 | For both |
| | $x = \frac{-12 - 2\sqrt{3}}{2(3 - 5\sqrt{3})} \times \frac{3 + 5\sqrt{3}}{3 + 5\sqrt{3}} \text{ oe}$ or $x = \frac{2 - 2\sqrt{3}}{2(3 - 5\sqrt{3})} \times \frac{3 + 5\sqrt{3}}{3 + 5\sqrt{3}} \text{ oe}$ with an attempt to simplify | M1 | For attempt to rationalise at least one of <i>their</i> solutions (must be similar structure) Sufficient detail must be seen, at least 3 terms in the numerator |
| | $\frac{1}{2} + \frac{\sqrt{3}}{2}$ | A1 | Must have sufficient detail shown |
| | $\frac{2}{11} - \frac{\sqrt{3}}{33}$ | A1 | Must have sufficient detail shown |

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| Question | Answer | Marks | Guidance |
|----------|---|-------|---|
| 3(a) | $b = \frac{1}{8}$ | B1 | |
| | $11 = a \sin \frac{4\pi}{8} + c$ $5 = a \sin \left(\frac{-4\pi}{3 \times 8}\right) + c$ | M1 | For attempt to form two simultaneous equations using given points, together with an attempt to obtain at least one unknown. Allow use of <i>their b</i> . |
| | a=4 | A1 | |
| | c = 7 | A1 | |
| 3(b) | Using symmetry | M1 | For e.g. period is 16π , symmetrical about the line $x = 8\pi$ |
| | For obtaining max at 4π and min at 12π | M1 | |
| | $x = 12\pi$ | A1 | |
| | y=3 | A1 | |
| | Alternative method 1 | | |
| | Minimum value when $y = 3$ | (B2) | FT on their $-a+c$ |
| | When $y = 3$, $x = 12\pi$. | (2) | M1 for attempt to solve <i>their</i> $3 = a \sin bx + c$ using <i>their</i> values of a , b and c to get $x =$ |
| | Alternative method 2 | | |
| | Min occurs $\frac{3}{4}$ through sine cycle so $x = 12\pi$ | (B2) | |
| | When $x = 12\pi, y = 3$ | (2) | M1 for attempt to solve $y = a \sin b (12\pi) + c$ using their values of a, b and c |
| | Alternative method 3 | | |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = ab\cos bx$ | (M1) | |
| | $(ab)\cos bx = 0$ | | |
| | $x = 4\pi, 12\pi$ | (M1) | Dep for attempt to solve to obtain <i>x</i> = |
| | $x = 12\pi$ | (A1) | |
| | y=3 | (A1) | cao |

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| Question | Answer | Marks | Guidance |
|----------|--|-------|---|
| 4(a) | $\frac{(2x-1)+4}{(2x-1)^2} = \frac{2x+3}{(2x-1)^2}$ | B1 | |
| | Alternative method | | |
| | $\frac{(2x-1)^2 + 4(2x-1)}{(2x-1)^3} = \frac{4x^2 + 4x - 3}{(2x-1)^3}$ $= \frac{(2x-1)(2x+3)}{(2x-1)^3} = \frac{2x+3}{(2x-1)^2}$ | (B1) | |
| 4(b) | Use of $\int \frac{1}{2x-1} + \frac{4}{(2x-1)^2} dx$ to obtain $\frac{1}{2} \ln(2x-1) - \frac{2}{(2x-1)}$ | 2 | B1 for $\frac{1}{2}\ln(2x-1)$ or equivalent B1 for $-\frac{2}{(2x-1)}$, allow unsimplified |
| | $\left[\frac{1}{2}\ln(2x-1) - \frac{2}{(2x-1)}\right]_{2}^{5}$ $\left(\frac{1}{2}\ln 9 - \frac{2}{9}\right) - \left(\frac{1}{2}\ln 3 - \frac{2}{3}\right)$ | M1 | For application of limits, must be in the form $a \ln(2x-1) + \frac{b}{(2x-1)}$ |
| | $=\frac{4}{9} + \ln\sqrt{3}$ | 2 | A1 for $\ln \sqrt{3}$ A1 for $\frac{4}{9}$ |
| 5(a) | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(3x \times \frac{4x}{\left(2x^2 - 3\right)}\right) - 3\ln\left(2x^2 - 3\right)}{9x^2} \text{oe}$ | 3 | B1 for $\frac{4x}{(2x^2-3)}$ M1 for attempt to differentiate a quotient or product A1 for all terms other than $\frac{4x}{(2x^2-3)}$ correct. |
| 5(b) | When $x = 2$, $\frac{dy}{dx} = 0.133$ | M1 | For substitution of $x = 2$ into their $\frac{dy}{dx}$ and use of h |
| | 0.133 <i>h</i> | A1 | |
| 5(c) | $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{4}{0.133}$ | M1 | For $\frac{4}{their \text{ value of } \frac{dy}{dx} \text{ from (b)}}$ |
| | 30.2 | A1 | |

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| Question | Answer | Marks | Guidance |
|----------|--|-------|--|
| 6 | $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sec^2 3x$ | 2 | M1 for $a \sec^2 3x$ |
| | When $x = \frac{\pi}{12}, y = 2$ | B1 | |
| | When $x = \frac{\pi}{12}$, $\frac{dy}{dx} = 6$ | M1 | |
| | Gradient of perpendicular is $-\frac{1}{6}$ | M1 | For $-\frac{1}{their} \frac{dy}{dx}$, must be numeric |
| | Equation of normal $y-2=-\frac{1}{6}\left(x-\frac{\pi}{12}\right)$ | M1 | For attempt at a normal equation using <i>their</i> $-\frac{1}{6}$ and 2 |
| | Area of triangle = 12 | 2 | M1 dep for attempt at correct area using <i>their</i> 2 and <i>their</i> $12 + \frac{\pi}{12}$ |
| 7 | $-\frac{1}{2}(2-3x)^{\frac{2}{3}}$ | 2 | M1 for $a(2-3x)^{\frac{2}{3}}$, $a \neq -\frac{1}{2}$ Allow unsimplified |
| | When $x = -2$, $\frac{dy}{dx} = -6$ leading to $c = -4$ | 2 | M1 Dep for correct attempt to find the value of the arbitrary constant |
| | $\frac{1}{10}(2-3x)^{\frac{5}{3}}$ nfww | 2 | M1 for $b(2-3x)^{\frac{5}{3}}$, $b \neq \frac{1}{10}$ Allow unsimplified |
| | When $x = -2$, $y = 10.2$ leading to $d = -1$ | M1 | Dep on previous M mark for attempt to find the value of a second arbitrary constant |
| | $y = \frac{1}{10}(2-3x)^{\frac{5}{3}} - 4x - 1$ | A1 | |
| 8(a) | $\begin{pmatrix} -40 \\ 42 \end{pmatrix}$ | B1 | Allow $2 \binom{-20}{21}$ |

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| Question | Answer | Marks | Guidance |
|-----------|--|-------|--|
| 8(b) | | B1 | FT on <i>their</i> answer to (a), must be numeric but not $\begin{pmatrix} -20 \\ 21 \end{pmatrix}$ |
| 8(c) | | M1 | Allow if in the incorrect order, FT on <i>their</i> (b), must have correct structure |
| | | A1 | |
| 8(d) | $AB = \sqrt{(5t-1)^2 + (2t+1)^2}$ | M1 | For attempt at modulus and square root using <i>their</i> answer to (c) |
| | $\sqrt{29t^2 - 6t + 2}$ | A1 | |
| 8(e) | $29t^2 - 6t - 4 = 0$ | M1 | For attempt to solve the square of <i>their</i> answer to (d) $-6 = 0$ |
| | 0.49 only | A1 | |
| 9(a)(i) | -0.4 | B1 | |
| 9(a)(ii) | $f(x) \in \mathbb{R}$ oe | B1 | |
| 9(a)(iii) | $x = \ln(5y+2) \text{ oe}$ $e^x = 5y+2 \text{ oe}$ | M1 | For a correct attempt to find the inverse |
| | $f^{-1}(x) = \frac{e^x - 2}{5}$ | A1 | Must be in the correct form |
| | $x \in \mathbb{R}$ | B1 | |
| 9(a)(iv) | y = f(x) $y = f(x)$ -0.2 0 0 0 0 0 0 0 0 0 0 | 4 | B1 for two correctly shaped graphs in the correct quadrants B1 for a correct graph for $y = f(x)$ with correct intercepts B1 for a correct graph for $y = f^{-1}(x)$ with correct intercepts B1 all correct with symmetry implied, exact intercepts and two points of intersection |

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| Question | Answer | Marks | Guidance |
|-----------|---|-------|---|
| 9(b) | $g^{2}(x) = \left(\left(x^{\frac{1}{2}} - 4\right)^{\frac{1}{2}} - 4\right)$ | M1 | For a correct order of operations |
| | $\left(\left(x^{\frac{1}{2}} - 4\right)^{\frac{1}{2}} - 4\right) = -2$ leading to $x^{\frac{1}{2}} = 8$, | M1 | Dep on previous M mark for a correct attempt at a solution. Must deal with $x^{\frac{1}{2}}$ correctly to obtain the final solution |
| | x = 64 | A1 | |
| 10(a) | Common difference = $4\sin 3x$ soi | B1 | |
| | $390 = \frac{20}{2} (2\sin 3x + 19(4\sin 3x))$ | M1 | M1 for attempt at sum to 20 terms using <i>their</i> common difference, equating to 390 and attempt to solve to obtain $\sin 3x =$ |
| | $\sin 3x = 0.5$ | A1 | |
| | $x = \frac{\pi}{18}, \ \frac{5\pi}{18}$ | 3 | M1 for a correct attempt to solve, may be implied by one correct solution, allow if not exact A1 for 1 correct solution A1 for a second correct solution and no others in the range |
| 10(b)(i) | Common ratio = $0.5 \cos y$ | B1 | |
| | $-0.5 < 0.5\cos y < 0.5$ | B1 | Correct use of common ratio < 1 |
| 10(b)(ii) | $9 = \frac{20\cos y}{1 - 0.5\cos y}$ | B1 | For attempt to use sum to infinity equation correctly and solve |
| | $\cos y = \frac{18}{49}$ or 0.367 | 2 | M1 for solution of <i>their</i> equation, must have r as a multiple of $\cos y$, to obtain $\cos y =$ |
| | 1.19 | A1 | |

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